Finding the Moment of Inertia for a Point Particle

For a point particle a distance r away from its axis of rotation, a force F will exert a torque r on the point particle. The magnitude of the torque exerted on the point particle is equal to

$$\tau = rF\sin\theta$$

if the angle between r and F is θ .

Let's say that θ is equal to 90 degrees, and so sin $\theta = 1$. The equation will then be:

$$\tau = rF(1)$$

$$\tau = rF$$

According to Newton's Second Law of Motion for translational motion, the net external force F acting on the point particle is equal to the mass of the particle m multiplied by its acceleration. If the force exerted on the point particle is at a right angle to r (we assumed it was when we said the angle between r and F is 90 degrees), then the applied force is a <u>tangential</u> force and is able to cause only a <u>tangential acceleration</u>, a_r for the particle, not a centripetal acceleration. Thus,

$$\Sigma \vec{F}_t = m \vec{a}_t$$

Let's make things even easier and say that only a single tangential force is being exerted on the particle:

$$\vec{F}_t = m\vec{a}_t$$

Let's forget about the direction of force and acceleration. If we need to, we can figure those out later on. Thus, we know that the magnitude of the tangential force acting on the point particle is equal to its mass multiplied by its tangential acceleration:

$$F_t = ma_t$$

Earlier, we said that a force F acting on a point particle a distance r away from its axis of rotation would exert a torque on the particle with a magnitude of r, and the magnitude of the torque can be found with the following equation:

if the angle between r and F is 90 degrees.

Because we assumed that θ = 90 degrees, the force F in the torque equation is the tangential force F_t acting on the point particle:

$$\tau = rF_t$$

We also found above that the magnitude of the tangential force F_t is equal to the mass of the point particle multiplied by the magnitude of its tangential acceleration a_t . Thus,

$$\tau = rma_t$$

We also know that the magnitude of the tangential acceleration a_t of the point particle is equal to the distance of the particle from its axis of rotation r multiplied by the magnitude of its rotational acceleration a_t .

$$\tau = rm(r\alpha)$$

If we simplify, we get

$$\tau = mr^{2}\alpha$$
$$\tau = I\alpha$$
$$I = mr^{2}$$

for a point particle.