Understanding and Expressing Vectors

Vectors can seem confusing to students when they first encounter them. So, let's talk a little bit about what a vector is and about how we can express vectors quantities.

1 WHAT IS A VECTOR?

The formal definition of a vector is as follows:

a **vector** is defined as a quantity that has both a **magnitude** and a **direction**. So, in order to understand vectors, we need to know what **magnitude** is and what **direction** is.

2 EXPRESSING VECTOR QUANTITIES NUMERICALLY

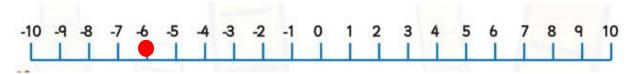
Magnitude can be understood as *size*, and there are various ways we can express size—or magnitude. One common way to represent magnitude is by using a *number with any necessary unit(s)*. For example,

- 3 meters is a magnitude because it has a number (3) and a necessary unit (meters),
- <u>5 meters/second</u> is a magnitude because it also has a number (5) and necessary units (meters/second),
- 20 degrees Celsius is a magnitude with a number (20) and a unit (degrees Celsius), and
- the number <u>7</u> is also a magnitude because it has a number. Note: Units are unnecessary here.

For me, **direction** is best understood using examples. Some examples of direction are as follows:

- north,
- south,
- east,
- west,
- up,
- down,
- left,
- right,
- etc.

But, direction can also be expressed in other ways. For instance, we can express direction as a **sign** (+/-). We can understand how sign (+/-) can be used to express direction by picturing the number line.



If I asked you where the red dot is located on the number line, you would probably easily tell me that the red dot is located at -6.

In the case of the location of the red dot, the *magnitude—the size—is 6*. We know that.

But, what does the minus sign (-) in front of the 6 mean about the location of the red dot? Of course we know that the minus sign in front of the 6 means that the red dot is located to the left of the origin (In this case, we are saying the origin is the o on the number line.). *Thus, the minus sign (-) gives us the direction of the position of the red dot relative to the origin*. In this case, the sign is the direction.

So, now we can put together what we know about magnitude and direction to understand what a vector quantity truly is and to understand how to express a vector quantity numerically. Let me restate the formal definition of a vector:

a **vector** is defined as a quantity that has both a **magnitude** and a **direction**.

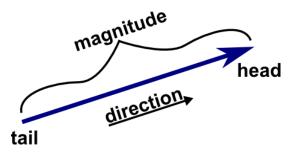
We now know that a magnitude can be expressed as a number with any necessary units, and we know what a direction is. Thus, a vector quantity can be expressed as **a number with any necessary units and a direction**. For example:

- 25 meters/second to the east is a vector quantity because it has a number with necessary units (25 meters/second) and a direction (to the east),
- **9.8 meters/second/second downward** is a vector quantity because it has a number with necessary units (9.8 meters/second/second) and a direction (downward), and
- -5 Newtons is a vector quantity because it has a number with necessary units (5 Newtons) and a direction (-).

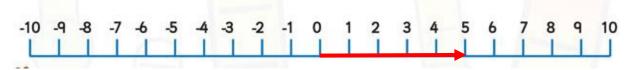
So, now we know that magnitude is **size**, and we know we can express size as a **number and necessary unit(s)**, and we know what a direction is. Thus, we know we can express any vector quantity **numerically**. In other words, we can express any vector quantity using a **number and necessary units** and a **direction**.

3 EXPRESSING VECTOR QUANTITIES GRAPHICALLY

Now we know how to express vectors numerically, but numerical representation is not always the most convenient way to express some vector quantities. We can alternatively express any vector quantity with a **drawing**. In math and science, we often refer to a drawing as a **graphical representation**. If we want to make a graphical representation of a vector—in other words, if we want to draw a vector—we simply draw it as an **arrow**. The **length** of the arrow represents the **magnitude**, or size, of the vector quantity being represented, and the **direction** the arrow points is the **direction** of the vector quantity.



For example, in the image below, the red arrow represents a vector with a magnitude of **5** and a **positive** direction (because it points to the positive side of the number line).



In the image below, the yellow arrow represents a vector with a magnitude of **6** and a **negative** direction (because it points to the negative side of the number line).



4 SUMMING IT ALL UP

In summary, we learned the following:

- 1. A **vector** is defined as a quantity that has both a **magnitude** and a **direction**.
- 2. Magnitude is size.
 - a. Vector magnitude—or size—can be expressed numerically as a number and necessary units, or
 - b. vector magnitude—or size—can be expressed graphically as the **length** of an arrow.

- 3. A **direction** is what we typically think it is and is generally understood well with examples.
 - a. Direction can be expressed numerically with one or more **words**, e.g., north, south, up, down, right, left;
 - direction can be expressed numerically with a sign (or even with a combination of signs, FYI), e.g., positive or negative (+/-); or
 - c. direction can be expressed graphically by drawing an **arrow** pointing in the direction of the vector quantity.

5 ADDITIONAL RESOURCES

You can find a great video introduction to vectors from the Khan Academy by clicking HERE.

You can watch the brilliant introduction to the villain named Vector in the movie Despicable Me by clicking <u>HERE</u>.

What are the magnitude and the direction of a vector?

Vectors. This is a **vector**: A **vector** has **magnitude** (how long it is) and **direction**: The length of the line shows its **magnitude** and the arrowhead points in the **direction**.

Vectors - mathsisfun.com www.mathsisfun.com/algebra/vectors.html

vec-tor

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noun

- MATHEMATICS PHYSICS

 a quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another.
- an organism, typically a biting insect or tick, that transmits a disease or parasite from one animal or plant to another.

verb

1. direct (an aircraft in flight) to a desired point.